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AN IMPROVEMENT OF FLOW FRONT COMPUTATION THROUGH BÉZIER SHAPE DEFORMATION GUARANTEEING MASS CONSERVATION LAW

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ABSTRACT: This paper aims to define the flow front as a continuous curve, in particular a Bézier curve, to reduce the inaccuracies generated by classical Finite Elements (FEM) in the flow front definition. The flow front is used in LCM processes for optimization algorithms, on-line control systems, PPI index and in general for whatever design and correction task. In these algorithms it is commonly used FEM simulation where the flow front is represented as a set of discrete points. This fact introduces an inaccuracy with the real flow front, because is continuous. In addition, the shape of the flow front obtained by FEM simulation differs in a great manner from the smooth shape of the real flow front. This concept was solved in our previous research, [7], where, using a mathematical technique, a Bézier curve is deformed and moved using velocity vectors. This technique is called Bézier Shape Deformation. This work improved the flow front representation but also introduced inaccuracies. In particular, the area enclosed between two Bézier curves, the flow front in different time instants, do not corresponds to the resins amount introduced in this range of time. To solve it, in the present research it is guaranteed the mass conservation law. Hence, it is introduced the required enclosed area in the mathematical technique to guarantee that not only velocity vectors has an influence in the Bézier curve deformation.

KEYWORDS: FEM, NEM, RTM, LCM, flow front, Bézier curves, numerical simulation.

1 INTRODUCTION.

The Liquid Composite Molding is the process most used in the production of aeronautic, naval industry, etc. This kind of process uses a mould to manufacture some pieces. After closing the mould is impregnated the preform (a special fiber) by injecting resin. The geometric line between the dry and wet area of the preform is defined as the flow front. It is a common tool to take decisions on-line during the filling of the mould. Usually numerical tools are used to simulate the flow front like Finite Elements Methods. This method introduces inaccuracies because the flow front is represented as a set of discrete points. The shape of the resin's flow obtained by FEM simulation differs in a great manner from the smooth shape of the real flow front. Actually it is a continuous curve. In general, the relevance of a proper representation of the domain was analyzed in other works like [1],[2],[3]. The idea is represent it as a continuous curve. The parametric curves are widely used for geometry description in CAGD (Computer Aided Graphic Design) because its mathematical properties are interesting. This fact has induced new numerical techniques to obtain a better representation of the computational domain with parametric curves, for examples, Bézier, B-Splines, NURBS, etc. In [4] introduced isogeometric analysis based on NURBS. The entire domain is defined as a NURBS surface where the simulation is computed. It produces an accurate representation of the flow front but with an inacceptable computational cost. To reduce it, in [5] is combined NURBS-Enhance Finite Element Method (NEFEM) with Discontinuous Galerkin Formulation, FEM. Only the mould contour is defined as a NURBS curve and inside the domain reducing the computational costs. This investigation improves the FE simulation results in the contour but not in the flow front because it is defined as a set of points like in FEM. In [6], the mesh of the domain is obtained by Bézier triangles. Although these works improve the FEM techniques, are not focused in the proper flow front representation. In this sense, it must be a continuous curve. This curve must be moved by velocity vectors obtained in the FEM simulation. This concept was developed in our previous research [7], where a Bézier curve was moved and deformed by velocity vectors.

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2 PREVIOUS WORK: BÉZIER SHAPE DEFORMATION. DEFINITIONS AND PROPERTIES.

Definition 1 *A Bézier curve of degree n can be represented as:*

$$\boldsymbol{\alpha}(t)=\sum_{i=0}^{n}\boldsymbol{P}_i B_{i,n}(t), t\in[0,1] \quad (1)$$

where $\boldsymbol{P}_i$ *are control points such that* $\boldsymbol{P}(0)=\boldsymbol{P}_0$ *and* $\boldsymbol{P}(1)=\boldsymbol{P}_n$, $B_{i,n}(t)$ *is a Bernstein polynomial given by:*

$$B_{i,n}(t)=\frac{n!}{i!(n-i)!}(1-t)^{n-i}t^i, i\in 0,1,...,n \quad (2)$$

Bézier curves have useful properties for represent a curve with a physical sense, in particular, an important properties is how to modify its shape. The way to get it, is modify the control points. With this properties it is defined the following operator:

Definition 2

$$\boldsymbol{S_\varepsilon}(\boldsymbol{\alpha}(t))=\sum_{i=0}^{n}(\boldsymbol{P}_i+\boldsymbol{\varepsilon}_i)B_{i,n}(t), t\in[0,1] \quad (3)$$

This definition represent the Bézier curve modify.This modification is made through velocity vectors. These velocity vectors are obtained by Finite Element Methods, joining the start point S and the final point T (Target Point), see Figure 1.

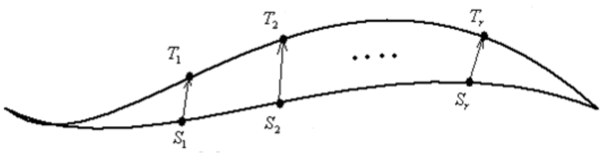

Figure 1: *Bézier Shape Deformation with vectors*

Another interesting properties is the easily way of concatenate two Bézier curves. It is necessary concatenate two or more Bézier curves to compute the real flow front using Bézier curve of low order. The aim of Bézier Shape Deformation (BSD) is to compute the vector $\boldsymbol{\varepsilon}$, it can be obtained through a constrained optimization method based on Lagrange Multipliers. The idea is to minimize the energy used by the curve from $\boldsymbol{\alpha}(t)$ to $\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}(t))$, that is,

$$\begin{aligned}&\min \parallel \mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}(t))-\boldsymbol{\alpha}(t)\parallel\\ &=\int_0^1(\boldsymbol{\alpha}(t)-\mathbf{S}_\varepsilon(\boldsymbol{\alpha}(t)))^2\,dt\\ &=\int_0^1(\sum_{i=0}^{n}\varepsilon_i B_{i,n}(t))^2\,dt\end{aligned} \quad (4)$$

To solve this problem, it is proposed the concatenation of multiple Bézier curves, k. Then, the function to minimize is translated as:

$$\sum_{j=1}^{k}\int_0^1(\sum_{i=0}^{n}\boldsymbol{\varepsilon}_i^{(j)}B_{i,n}(t))^2\,dt \quad (5)$$

To maintain the derivative property of the curve is imposed to the constrained optimization method the derivative restrictions in the start and end point of the resulting concatenated curve,

$$\boldsymbol{\alpha}_1'(0)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_1(0))=0;\boldsymbol{\alpha}_k'(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_k(1))=0 \quad (6)$$

And in the joined points of the concatenated curves,

$$\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}_1(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}_2(0))=0 \quad (7)$$

$$\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_1(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_2(0))=0 \quad (8)$$

...

$$\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}_{k-2}(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}_{k-1}(0))=0 \quad (9)$$

$$\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_{k-2}(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_{k-1}(0))=0 \quad (10)$$

The Lagrange function can be defined as:
$L(\boldsymbol{\varepsilon}_i,\boldsymbol{\lambda})=$

$$\begin{aligned}=\;&\sum_{l=1}^{k}\int_0^1(\sum_{i=0}^{n_1}\varepsilon_i^{(l)}B_{i,n_k}(t))^2+\\ &+\sum_{j=1}^{r_l}\boldsymbol{\lambda}_j^{r_l}[T_j^{(l)}-\mathbf{S}_{\boldsymbol{\varepsilon}}(\alpha_l(t_j))]+\\ &+\boldsymbol{\lambda}^{(2l-2)-1}[\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}_{l-1}(1))-\mathbf{S}_{\boldsymbol{\varepsilon}}(\boldsymbol{\alpha}_l(0))]\\ &+\boldsymbol{\lambda}^{(2l-2)}[\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_{l-1})(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_l(0))])\\ &+\boldsymbol{\lambda}_s[\boldsymbol{\alpha}_1'(0)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_1(0))]\\ &+\boldsymbol{\lambda}_{s+1}[\boldsymbol{\alpha}_k'(1)-\mathbf{S}_{\boldsymbol{\varepsilon}}'(\boldsymbol{\alpha}_k(1))]\end{aligned} \quad (11)$$

Making zero $\frac{\partial L}{\partial \boldsymbol{\varepsilon}}$ and $\frac{\partial L}{\partial \boldsymbol{\lambda}}$ a linear system equations is obtained like $AX=b$, where A is a square matrix, its dimension depends on the order of the matrix. The vector X is the solution and with it the Bézier curve is modified.

3 OBJETIVE AND OUTLINE

The "Bézier Shape Deformation" improved the flow front representation but as well introduced inaccuracies. One of the inaccuracies is a problem of the Bézier curves. Depending on the geometrical situation of the control points, in some situations the curve has a loop, see [8]. To compute the real flow front is necessary avoid this problem, see Figure 2.

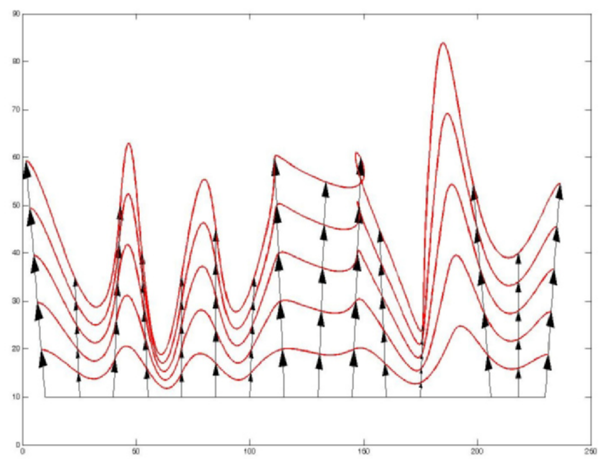

Figure 2: *The flow front representation with loops.*

Another inaccuracies presented in the BSD, is the area enclosed between two Bézier curves.This area does not correspond to the resin's amount introduced in a range of time. To solve it, in the present research it is guaranteed the mass conservation law. Hence, it is introduced the required enclosed area in the mathematical technique to guarantee that not only velocity vectors has an influence in the Bézier curve deformation. At the end of the paper some examples are shown.
The mathematical technique to develop this tool is the same like in the "Bézier Shape Deformation", a constrained optimization method based on Lagrange multipliers. Only a new constraint is added that guarantees an equality between the area and the resin's amount injected.

4 THEOREM OF GREEN-RIEMANN

Theorem 1 *Let $\beta : [a,b] \longrightarrow \mathbb{R}^n$ Jordan's curve piecewise smooth. Let D a convex set bounded by β. β is positively oriented. Let $F = (M,N)$ vector field, $F : A \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $F \in C^1(A)$ such that $D \subset A$. Then,*

$$\int_\beta M\,dx + N\,dy = \int\int_D (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})dA \qquad (12)$$

Corollary 2 *A consequence of Green-Riemann Theorem is used to compute the area of D as:*

$$\text{area}(D) = \int\int_D dA = \int_\beta -y\,dx \qquad (13)$$

This result, corollary 2, is used to compute the area enclosed two Bézier curves. In our paper, it is used to compute the area between two flow front's representation in different time instants. This value is compared with the resin's volume injected in this time instant. See Figure 3.

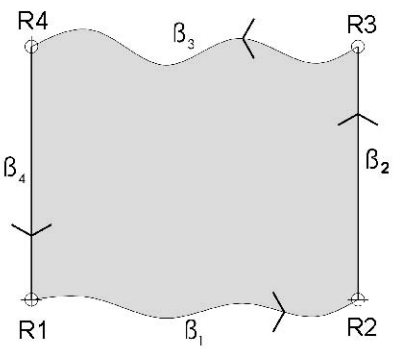

Figure 3: *Area between two Bézier curves*

Definition 3 *Let $F : A \subset \mathbb{R}^n \longrightarrow \mathbb{R}^n$ a continuous vector field and let $\gamma : [a,b] \longrightarrow \mathbb{R}^n$ piecewise smooth curve. The line integral along γ is defined as follow:*

$$\int_\gamma F = \int_a^b F(\gamma)\cdot\gamma'(t)\,dt \qquad (14)$$

5 BÉZIER SHAPE DEFORMATION INTRODUCING A PHYSICAL LAW AND WITHOUT LOOPS

To develop this new technique both problems have been solved: the loops and the are enclosed. To solved the loops in the Bézier curve it had been reduced the order.The objective is an injective Bézier curve, the option to get it is reduce the order until second order and concatenate "k" Bézier curves to represent the full resin's flow. If the area enclosed between to Bézier curves must be the resin's amount introduced in a range of time, then a new constraint is added in the Lagrangian function, see 11. The new Lagrangian function will be:

$$\overline{L(\boldsymbol{\varepsilon},\boldsymbol{\lambda})} = L(\boldsymbol{\varepsilon},\boldsymbol{\lambda}) + \boldsymbol{\lambda}(area - amount) \qquad (15)$$

The area's computation is done with the corollary 2. With this mathematical result is possible to calculate the area with the boundary's parametrization. In this case, see Figure 3, the parametrization is known because the flow front is a Bézier curve and the contour of the mold in this previous research is considered as a straight line. In figure 3 there are four curves, $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, $\boldsymbol{\beta}_3$ and $\boldsymbol{\beta}_4$. The first and the third one are a concatenated Bézier curves, its parametrization is defined in equation (1). The second and the fourth one are segments. The parametrization of these segments are defined as follows:

$$\boldsymbol{\beta}_2 = (x_2(t), y_2(t)) = \mathbf{R}_2 + t\overrightarrow{\mathbf{R}_2\mathbf{R}_3} = \mathbf{R}_2 + t(\mathbf{R}_3 - \mathbf{R}_2) \qquad (16)$$

$t \in [0,1]$

$$\boldsymbol{\beta}_4 = (x_4(t), y_4(t)) = \mathbf{R}_4 + t\overrightarrow{\mathbf{R}_4\mathbf{R}_1} = \mathbf{R}_4 + t(\mathbf{R}_1 - \mathbf{R}_4) \qquad (17)$$

$t \in [0,1]$

Applying the definition 3 the area of this zone is compute as: (considering $\boldsymbol{\beta}(t) = (x(t), y(t))$)

$$\text{area}(D) = \int_{\boldsymbol{\beta}} -y\,dx = \int_0^1 F(\boldsymbol{\beta}(t))\cdot\boldsymbol{\beta}'(t)\,dt = \qquad (18)$$

$$= \sum_{i=1}^{4}\int_{\boldsymbol{\beta}_i} -y\,dx = \sum_{i=1}^{4}\int_0^1 -y_i(t)\cdot x_i'(t)\,dt$$

The result of this area is a function with the control points $\mathbf{P}_i$of the Bézier curves and another function with the perturbation vector $\boldsymbol{\varepsilon}_i$. Making zero $\frac{\partial L}{\partial \boldsymbol{\varepsilon}}$ and $\frac{\partial L}{\partial \boldsymbol{\lambda}}$ is obtained a no linear system, this is the difference between this method and the Bézier Shape Deformation. The system is $F(X) = 0$ and the solution is $\mathbf{X} = [\boldsymbol{\varepsilon}_i, \boldsymbol{\lambda}]$. To solve this non-linear system, the method used is "trust region-dogleg", a numerical method to solve non linear systems. The solution is a better representation of the flow front.

6 EXAMPLES

In order to show the numerical results of the presented technique, some examples are shown. The first example, see Figure 4, is the proof that the method works using parallel vectors with the same length and knowing the area's value between the Bézier curves. The second one is an example of the deformation of two Bézier curves changing the angle and the length of the vectors, see Figure 5.

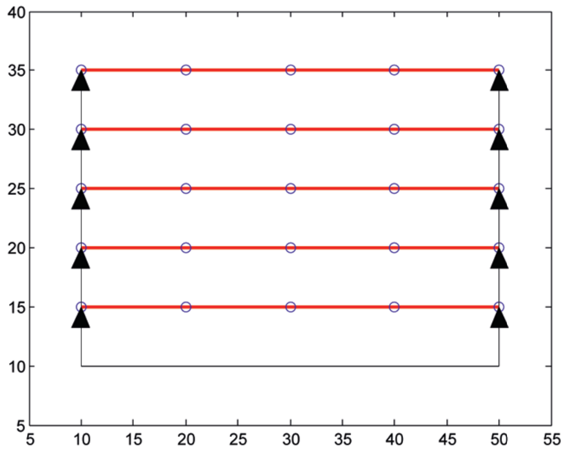

Figure 4: *Example1*

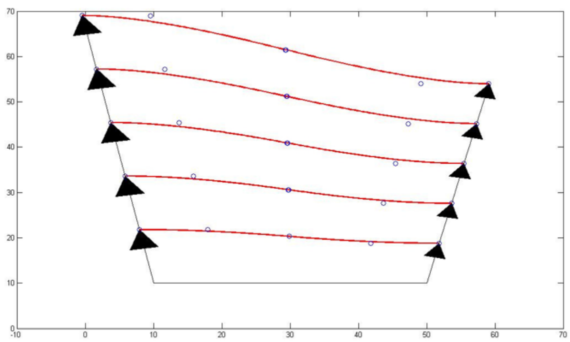

Figure 5: *Example2*

7 CONCLUSIONS

In a previous work was developed a new mathematical technique to get a continuous representation of the flow front, in addition to this the curve could be updated with a vector displacement field.It was the first study using CAGD techniques to represent the flow front as a continuous curve. Now, in this research the solution have been improved avoiding the loops and including the resin's amount information. The idea is to get an ideal representation of the flow front better than the solution obtained during simulation. The inconvenient of this method is the system obtained, with the BSD the solution was obtained with a linear system. In this case, the solution is obtained with an iterative method because it is a non linear system.

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